387

VOLUME 80 SEPARATE No. 387

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

JANUARY, 1954

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ENGINEERING MECHANICS
DIVISION

{Discussion open until May 1, 1954}

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

AN EXPERIMENTAL STUDY OF BUBBLES MOVING IN LIQUIDS

W. L. Haberman[1] and R. K. Morton[2]

SYNOPSIS

In connection with other investigations at the David Taylor Model Basin a fundamental study of the motion of bubbles was undertaken. As an initial step, experiments were conducted to determine the drag and shape of single air bubbles rising freely in various liquids.

The results of the experiments show that a complete description of the motion of air bubbles by use of three dimensionless parameters containing the usual physical properties of the liquid (viscosity, surface tension, density) is not possible. Three types of bubble shapes were observed in each liquid, namely spherical, ellipsoidal, and spherical cap. For a specific liquid the shape of the bubble was a function of its volume.

For tiny spherical bubbles the drag coefficients coincide with those of corresponding rigid spheres. With increase in bubble size, a decrease in the drag as compared to that of rigid spheres occurs in some liquids. Thus, the drag curves of the spherical bubbles rising in various liquids fall between two limiting curves, namely the drag curve of rigid and fluid spheres, respectively. It was not possible to determine a criterion for the transition of the bubbles from rigid to fluid spheres. Ellipsoidal bubbles occur at different ranges of Reynolds numbers for the various liquids. The drag coefficients of spherical cap bubbles are independent of bubble size and have a constant value of 2.6.

For bubbles (equivalent radius 0.03 to 0.30 cm) rising in tap water or in water containing certain surface-active substances, experiments show an increase in drag as compared to bubbles in pure water.

Results of tests to determine the effect of the container walls on the velocity of rise are also presented and a description of the experimental apparatus is given.

INTRODUCTION

A detailed knowledge of the mechanism of bubble motion is of interest to the many fields of engineering which deal with the prediction of performance of equipment and of processes involving the motion of gaseous bodies. In general, the equation of motion of such bodies is given by:

Drag + Pressure Force + Weight = (Mass + Added Mass) x Acceleration

This equation can be solved if the total mass of the bubble and all forces acting on it are known.

The drag of a bubble is, in general, a complicated function of its geometry, its velocity, and the physical properties of the medium. The shape which the

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bubble assumes is in turn a complex function of the hydrodynamic, viscous, and interfacial forces exerted. In addition, such effects as container walls and bottom, free surface of the liquid and the effect of adjacent bubbles may also have a pronounced influence on the drag of a bubble. Unfortunately, there is only meager information available on the drag of gas bubbles and practically none on their added masses.

As an initial step to obtain information on the drag of gas bubbles, a study of single bubbles rising freely at their terminal velocity in various liquids was undertaken. For the case of motion at such constant velocity and above equation reduces to:

$$\text{Drag} + \text{Buoyant Force} + \text{Weight} = 0$$

since all forces acting on the bubble are in equilibrium. Specifically, the experiments described in this paper consisted of the determination of the terminal velocity, shape, and path of single air bubbles rising freely in various liquids as a function of bubble size. The possible effect of the walls of the container on the velocity of rise was also investigated. Future studies of the effect of different pressure gradients on the drag and studies of apparent masses of gas bubbles are contemplated.

Theoretical solutions for the drag of bodies, applicable to gas bubbles, have been obtained for spheres moving slowly in an infinite medium.

For rigid spheres: $D = 6\pi\mu\, rU$ Stokes (1)

For fluid spheres: a) $D = 4\pi\mu\, rU$ Hadamard, Rybczynski (2)

b) $D = 6\pi\mu\, rU\, \dfrac{e + 2r\mu}{e + 3r\mu}$ Boussinesq (3)

where D is the drag of the sphere, μ the coefficient of dynamic viscosity of the liquid medium, r the radius of the sphere, U the terminal velocity of the sphere, and e the coefficient of surface viscosity. For the fluid spheres (in contrast to the rigid case) non-zero tangential velocities at the sphere interface are assumed. Hadamard and Rybczynski assumed equality of the tangential velocities and stresses at the bubble-liquid interface, while Boussinesq included changes in the interfacial stresses due to 'dynamic' surface tension. In the last of the above equations, the factor $\dfrac{e + 2r\mu}{e + 3r\mu}$ approaches 1 for r approaching zero or e very large; it approaches 2/3 for $e \angle\angle r\mu$, i.e. for large r or for e approaching zero. Hence, for very small bubbles, Boussinesq's solution approaches Stokes' law as a limit, while the other limit is the Hadamard-Rybczynski solution.

A dimensional analysis of the motion of rising bubbles yields when considering eight physical variables[3];

$$f_1\ (C_d,\ R,\ W,\ \mu/\mu',\ \rho/\rho') = 0$$

or $$f_2\ (C_d,\ R,\ M,\ R',\ \rho/\rho') = 0$$

3. These physical variables are: the velocity of the bubble (U), the acceleration due to gravity (g), the density of the liquid medium (ρ), the density of the bubble (ρ'), the length parameter of the bubble (ℓ), the coefficient of dynamic viscosity of the liquid (μ) and of the gas (μ'), and the surface tension (σ).

and so on, where C_d is the drag coefficient, R the Reynolds number $\left(= \frac{2\ell U \rho}{\mu}\right)$, W the Weber number $\left(= \frac{2\ell U^2 \rho'}{\sigma}\right)$, R' the Reynolds number inside the bubble $\left(= \frac{2\ell U \rho'}{\mu'}\right)$, and M a dimensionless parameter $\left(= \frac{g\mu^4}{\rho\sigma^3}\right)$. If the density and viscosity of the gas are considered negligible, the number of dimensionless groups are reduced to three:

$$f_3 (C_d, R, W) = 0$$

or $$f_4 (C_d, R, M) = 0$$

or $$f_5 (C_d, W, M) = 0$$

etc. For bubbles, the most convenient length parameter is the equivalent radius (r_e), defined as $\sqrt[3]{\frac{\text{volume}}{(4/3)\pi}}$. The drag coefficient then becomes $\frac{8}{3}\frac{gr_e}{U^2}$. For the Stokes and Hadamard-Rybczynski solutions the drag coefficients are, respectively

$$C_d = 24/R \quad \text{(Rigid sphere)}$$

$$C_d = 16/R \quad \text{(Fluid sphere)}$$

For the special case in which only the velocity, the acceleration of gravity, the density of the liquid, and the equivalent radius are considered, one dimensionless parameter is obtained, namely C_d = constant. This solution will be shown to apply to the region of very large (spherical cap) bubbles.

Previous experimental work on the rising motion of bubbles has been extensive. The investigations have, however, largely been restricted to bubbles rising in water. A detailed bibliography of these investigations may be found in Reference (4).

Experimentation

The experimental study consisted of measuring the terminal velocity of individual bubbles of various sizes rising in eight liquids. It also included the determination of the effect of the walls of the container on the bubble velocity. Details of the experimental apparatus and procedure, the generation of the bubbles, and the test liquids are given in the following paragraphs.

Test Tanks and Liquids. - The tests were performed in three transparent wall tanks; the large one was of 3 x 3 ft cross section and 5 ft. height, the medium one of 1 x 1 ft. cross section and 3 1/2 ft. height, while the small one was of 6 x 6 inches cross section and 2 ft. height. In addition, tests were also performed in an insert of 6 x 6 inches cross section and 20 inch height placed in the center of the medium tank.

Since the large tank was of sufficiently large dimensions, no appreciable wall effect was expected.[4] The medium tank was chosen of dimensions large

4. A few of the previous TMB tests (5) were repeated in the large tank to observe any change in results. These previous tests were conducted in a tank of 4 1/2 x 25 ft. cross section and 9 ft. height with 8 ft. depth of filtered water at room temperature, using one end of the tank for the tests.

enough to reduce wall effects, yet small enough to allow use of a variety of liquids. The small tank and insert provided an additional tank size. It was intended, if wall effect existed, to extrapolate the results obtained in the finite containers to the case of an infinite medium.

The eight test liquids were water (at three different temperatures), Varsol[5], methyl alcohol, water containing 0.42% (by volume) Glim[6], turpentine, mineral oil, and two corn syrup-water mixtures. Turpentine was selected as one of the test liquids because it has, at room temperature, the same viscosity as cold water. One of the corn syrup mixtures had approximately the same viscosity as the mineral oil. The viscosity of the liquids was measured by means of ordinary and modified Ostwald viscosimeters. The accuracy of measurement of viscosity was 1 1/2% and 1/2%, respectively. The surface tension was determined by the capillary-rise method (accuracy of measurement: 3%) and the specific gravity of the liquids was obtained by means of hydrometers (accuracy of measurement: 1%). These physical properties were measured following the completion of each test. They are summarized in Table 1 together with those of liquids used by several other investigators. (6, 7, 8).

In Varsol and water (room temperature and hot), tests were conducted in all three tanks; for cold water and mineral oil, in the medium tank and insert; and for all other liquids, in the small tank only.

Test Procedure. - Small bubbles were generated by means of hypodermic needles and glass nozzles of various sizes. The larger bubbles were obtained by use of a dumping cup, which was inverted to release the air bubble. The nozzles and needles were connected to a brass tube which was fastened to a sliding mechanism (Figure 1). This sliding mechanism allowed the tips of the various nozzles to be placed at the identical position, eliminating the need for refocusing the camera after each change of nozzle. The air was supplied from a compressed air bottle. A needle valve regulated the air flow so that bubbles were released at the interval desired.

The bubble size was determined by 'weighing' a sufficient number of bubbles in the inverted funnel (Figure 1) by means of an analytical balance. Since the density of air is negligible in comparison to that of a liquid, the difference in balance reading equals the buoyancy of the bubbles (i.e. it equals the volume of the bubbles times the density of the liquid). The change in balance reading was always at least 0.2 gm, resulting in an accuracy of measurement of volume of 1%. The volume of the individual bubble was obtained by dividing the total volume by the number of bubbles collected in the funnel. A comparison of photographs of different bubbles showed that the bubble size did not vary if the frequency of bubble generation remained constant. Large bubbles from the dumping cup were weighed individually. The volume of the bubble was adjusted for the change in pressure due to difference in depth between the level at which the rate of rise is determined and the level of the inverted funnel. This was done by use of the general gas law at constant temperature taking into account the partial pressure of the saturated vapor at test temperature.[7] Tiny spherical bubbles could not be generated at a frequency to

5. A trade name (Standard Oil Company) for mineral spirits (heavy naphtha), a petroleum derivative.
6. Glim, a surface active agent, is the trade name (B. T. Babbitt, Inc) of a non-ionic, liquid detergent, a condensation product of ethylene oxide and lauryl alcohol.
7. Details of this correction may be found in References (4) or (5).

allow a sufficient number to be collected in the funnel, hence their size was determined from the photographic record. No correction for change in depth is then needed.

To avoid any changes in the volume of the bubble due to air interchange with liquid, the latter was saturated with air prior to actual testing. This was accomplished by stirring the liquid and by blowing air through it. The liquids for all but the cold and hot water tests were at room temperature, which varied little throughout the day. Water was cooled by circulation through a water cooler; it was heated by immersion heaters or obtained directly from the hot water faucet. Both filtered and tap water were used in the tests.

Uniformity of liquid temperature was achieved by means of mechanical stirring before each test. Frequent checks of temperature at various locations inside the tank were made by means of immersion thermometers. In the process of stirring, small bubbles appeared in the liquid. The irregularity of motion of these small bubbles, which were still present after completion of the stirring, served as an indication of the presence of residual turbulence in the liquid. In sufficient time, the motion of the small bubbles always became regular and hence indicated that the residual turbulence, if still present, was not large enough to affect the motion of the bubbles. The actual test was not begun until all of the small bubbles reached the surface of the liquid.

The rate of bubble flow was then regulated by the needle valve so as to release bubbles with a minimum spacing of 24 inches. This reduced the effect of the wake created by the passage of a bubble on the motion of a bubble following. The same precaution was observed for the larger bubbles that were formed by dumping. An additional precaution was to rotate the dumping cup with steady speed in order to avoid splitting the bubble or the formation of satellites upon release. The slow passage of the air through the brass tube inside the tank allowed the air to reach the temperature of the liquid. Contact of the air at the nozzle tip or inside the dumping cup with the liquid allowed saturation of the air with liquid vapor, so that the air bubble can, in each instance, be assumed to be saturated with the vapor of the liquid in which it rises.

Motion Pictures and Their Evaluation. - The velocity, path, and shape of the bubbles were obtained from motion pictures made with a Mitchell 35mm camera using a special lens attachment to permit close-ups. Film speeds of 25 to 35 frames per second and back lighting from a white reflector were used. For the first few tests, the film speed was obtained by photographing a rotating clock dial; subsequently a neon timing light (located inside the camera) with a 60-cycle voltage source was utilized. From the marks of the timing light on the film, the film speed was determined. The field of the camera ranged from 1.4 x 1.8 to 1.75 x 2.3 inches depending upon the refractive index and horizontal depth of the liquid. A transparent scale photographed in the plane of the bubble provided the distance scale factor for the evaluation of displacement and size. The camera lens was placed at approximately the midpoint between liquid level and bottom of tank for all tests. The camera location was in each instance located sufficiently above the nozzle tip so that the bubbles reached their terminal velocity before passing in front of the camera.

Changes in bubble volume due to differences in liquid depth were minimized by making velocity measurements over a very short vertical displacement (less than 2 1/2 inches). The rate of rise of bubbles was determined by measuring the displacement of a bubble from a reference point on successive

frames of the film by means of a Bausch and Lomb contour measuring projector using a magnification of twenty-five. These displacements were then plotted against the frame number. A straight-line plot indicated that the velocity of the bubble remained constant during the time it passed the field of the camera. From the slope of the line, the frame speed, and scale factor, the velocity of the bubble was computed.

Results

Terminal Velocity of Bubbles. - The results of tests to determine the velocity of rise of gas bubbles in various liquids are most conveniently presented as a function of the equivalent radius of the bubble, defined as the radius of a sphere having the same volume as the bubble. Typical results are shown in Figures 2 and 3 for filtered water (including data from other investigators) and for tap (unfiltered) water. A summary of all TMB velocity curves (except those for tap water) and those obtained by Bryn is given in Figure 4. A compilation of the properties of the liquids was given in Table 1. In general, the results as seen from Figure 4 indicate that for small (spherical) bubbles of given volume the viscosity of the liquid is the most important property determining the rate of rise. Very large bubbles (spherical caps) rise independently of the properties of the liquid.

The results of tests to determine the effect of the container walls on the velocity of a bubble, conducted in tanks of different sizes in water, Varsol, and mineral oil, showed within the limits of experimental accuracy the absence of any wall effect for the range of bubble sizes tested. Figures 2 and 3 are typical of these results. Subsequent tests in the other liquids were made in the small tank only and its results may be applied to the case of an infinite medium.

No systematic investigation of vertical proximity effect was made, i.e. the effect of the wake created by the passage of a bubble on the motion of a bubble rising at a distance below. Such effects were avoided in the experiments by sufficiently spacing the bubbles. However, the results of a few special observations indicate that proximity effects may be appreciable. For example, tests in mineral oil show an increase of 9% and 39% for bubbles of equivalent radius of 0.17 cm, rising 7.7 cm and 3.2 cm apart respectively. The presence of the wake in the liquid thus results in higher velocities of rise of the bubble.

Figure 5 compares the rate of rise of air and of oxygen bubbles in distilled water. No significant change in the rate of rise of the bubbles with change in the gas composing it is indicated. It is, therefore, reasonable to assume that the velocity curves for bubbles composed of various other gases would be identical to those for air bubbles.

Nondimensional Evaluation. - The results of the TMB bubble tests and those of Arnold, Bond and Newton, and Bryn were plotted in terms of three dimensionless products. Figure 6 presents the results in terms of drag coefficient, Reynolds number, and the parameter 'M', while Figure 7 presents them in terms of the drag coefficient, the Weber number, and 'M',[8] The curve for filtered or distilled water at a temperature of 19°C was drawn through points obtained from the experiments of Bryn and the TMB tests. Examination of Figure 6 or 7 shows no systematic arrangement of the curves

8. The results for tap water and for water containing Glim are not shown. They will be discussed in subsequent sections.

with change in the parameter M, which is constant for a specific liquid. It can, therefore, be concluded that neither of the nondimensional sets presented nor any other complete set using the same six variables (namely velocity, acceleration of gravity, density and viscosity of the liquid, surface tension, and equivalent radius) is sufficient for complete description of bubble motion.

The question now arises whether correlation of bubble data could be obtained by using two additional dimensionless parameters, for example the liquid to air viscosity and density ratios or the Reynolds number inside the bubble and the density ratio etc. The results of the experiments conducted do not permit conclusions regarding the importance of these parameters. A short discussion of the significance of the internal Reynolds number will be given in subsequent sections.

Three types of bubble shapes were observed in the experiments. Very small bubbles are spherical. Larger bubbles are flattened, i.e. ellipsoidal in shape, whereas very large bubbles assume a spherical cap shape. Of course, the volumes at which these transitions occur vary with the liquids. Photographs of typical shapes are shown in Figure 8. It should be noted here that some of the shapes shown in these photographs are instantaneous shapes, since the shape of large bubbles does not remain constant during the ascent, except for those bubbles rising in a highly viscous medium (e.g. mineral oil and corn cyrup).

Table 2 shows the approximate extent of the three shape regions as a function of Reynolds number for the various liquids tested at TMB. From this table and Figure 6 (which also includes the drag curve for rigid spheres (11) and the lines for the Stokes and the Hadamard-Rybczynski law) the following can be observed: The drag curves of spherical bubbles in the various liquids fall between two limiting curves. As an upper limit, the drag curve of rigid spheres is obtained, while the lower limit is the drag curve for fluid spheres. With decreasing Reynolds number the rigid sphere curve connects with the straight line of Stokes' Law, while the fluid sphere curve connects with the line of Hadamard-Rybczynski's Law. The curve for the fluid spheres was obtained by drawing the lower envelope to the experimental curves; its accuracy can be confirmed by additional tests in other liquids or by extension of the theoretical solution into regions beyond that of very slow flow.

It will be also noted from Figure 6 that the curve for mineral oil, for example, follows the straight line of Hadamard-Rybczynski's law over a certain region of Reynolds numbers. This indicates that the boundary conditions assumed in the analytical solutions for fluid spheres are actually fulfilled and that circulation exists inside the bubble. The experimental curves also indicate an interesting aspect of the phenomenon of bubble motion, namely that with decreasing Reynolds number, the drag coefficient of the bubbles becomes equal to the drag of rigid spheres. The transition may occur at a Reynolds number of about 40 (as for filtered and distilled water) or may not take place until very low Reynolds numbers are reached, i.e. well within the region of slow flow (as for olive oil (7) or very viscous syrup (6)). Thus, from the experimental data available it appears certain that tiny air bubbles rising in any liquid follow Stokes' Law.

For bubbles behaving like rigid bodies, thus indicating absence of motion inside the bubble, the internal Reynolds number (although non-vanishing) is of no significance in describing the rising motion of the bubbles. Likewise, the internal Reynolds number cannot be used to predict the transition point at which the drag of the bubbles becomes less than that of corresponding rigid spheres. Beyond this transition point, the internal Reynolds number might be of importance in the description of the motion of the bubbles.

Surface tension tends to make the surface area of the bubble as small as possible. For a given volume, the configuration of minimum surface area is a sphere. This effect of surface tension would be most pronounced for bubbles of small radii. For larger bubble sizes, the surface forces become smaller in comparison to the viscous and hydrodynamic forces and flattening of the bubble occurs. This flattening to approximately an oblate spheroid results in higher drag as compared to a sphere of the same volume. From Figure 6 and Table 2, it will be observed that the region of ellipsoidal bubbles for the various liquids occurs at different ranges of Reynolds number; that for liquids of low 'M' number a minimum in the drag curves is reached at Reynolds numbers of the order of 250; and that these minima occur near the transition from spherical to ellipsoidal shape. Such minima are not obtained for liquids of high 'M' number. The drag coefficients of bubbles in such liquids decrease until a constant value for the drag coefficient (spherical caps) is attained. In the ellipsoidal region the curves are arranged according to the magnitude of the 'M' number, indicating that the liquid properties contained in this parameter, namely surface tension, viscosity, and density, are of primary importance in the motion of these bubbles. Thus, in this region the Reynolds number inside the bubble is of no importance in the description of bubble rise, since correlation was obtained in terms of drag coefficient, Reynolds number, and 'M' number.

When the viscous and surface tension forces become small relative to the hydrodynamic forces, the shape assumed by the bubbles is that of the so-called spherical caps. Typical shapes of these bubbles are shown in Figure 8. The upper surface is essentially spherical, while the lower surface varies from a highly irregular one for liquids of low viscosity to a smooth surface for very viscous liquids. The configuration of the upper surface results almost exclusively from the hydrodynamic forces.[9] The results of the present tests in a number of liquids confirm the constant value of 2.6 for the drag coefficient obtained by previous investigators (5, 12). The velocity ofspherical cap bubbles of given size rising in any liquid can be determined from the value of the drag coefficient or directly from the velocity curve (Figure 4). For $C_d = 8/3/gr_e/U^2 = 2.6$, we obtain for the rate of rise of the spherical caps in all liquids

$$U = 1.02 \sqrt{gr_e}$$

Path of Rising Bubbles. - Figure 9 shows typical paths and corresponding shapes of bubbles in four liquids. Three types of motion of the bubbles were observed in the experiments: (1) rectilinear motion (2) motion in a helical path and (3) rectilinear motion with rocking. The motion of spherical bubbles is either rectilinear or helical. For ellipsoidal and spherical cap bubbles all three types of motion can occur. It appears that the type of motion may be predicted from the value of the Reynolds number at which the motion takes place. Below Reynolds numbers of about 300 the motion is rectilinear. With increase in Reynolds number spiraling begins and increases in amplitude and frequency until a maximum is reached. At Reynolds numbers of about 3000 the spiraling disappears and only rectilinear motion with rocking is obtained. For the bubbles rising in mineral oil and the corn syrup mixtures, only rectilinear motion without rocking was observed. The maximum Reynolds number reached during those tests was 150.

9. See e.g. Reference (12).

The helical path of the bubbles assumed either a clockwise or counter-clockwise direction, depending upon conditions at generation. The velocity of rise of these bubbles is not affected by the sense in which the bubble revolves. The major axis of ellipsoidal bubbles is always perpendicular to the direction of motion.

The oscillatory motion of bubbles is probably caused by the periodic shedding of vortices behind the bubble. Such vortex shedding has been observed experimentally for rigid spheres at the same magnitudes of Reynolds numbers as for bubbles (13).

Bubbles in Tap Water and in Water Containing Surface-Active Substances. - The drag coefficients for air bubbles rising in tap water at two different temperatures are given in Figure 10. Gorodetskaya's (14) results in tap water at room temperature are also included. For comparison, the drag curves for bubbles in filtered and distilled water at room temperature and for rigid spheres are also shown. In the region of spherical and spherical cap bubbles, the drag curves at the two temperatures coincide. The value of the minimum drag coefficient is, however, greater than that of the corresponding filtered water. In general, for Reynolds numbers up to about 300, the drag curves of bubbles in tap water follow closely the curve of rigid spheres.[10] Thus, the results of the experiments show that for bubbles (ranging in equivalent radius from 0.035 to 0.25 cm) it is important whether the motion occurs in filtered (distilled) or tap water.

In view of the fact that merely filtering the water was sufficient to produce a change in the drag of the bubbles, it is indicated that the presence of minute particles causes this change. Most of these particles, which are known to exist in ordinary tap water, can be removed by filtering. Specifically, if such particles are present, a high concentration of them would be found at the surface of the bubble. The particles at the surface would travel with the bubble, hence imparting, in effect, a rigid surface to the bubble. With increasing bubble velocities, the shear forces become large in comparison to the forces holding the particles to the surface and hence at a certain critical velocity no high concentration of particles on the surface can exist.

The effect of surface-active substances on the rate of rise of air bubbles has previously been investigated by Gorodetskaya (14), who added small concentrations of various alcohols to water and concluded that, beyond a certain critical concentration of the surface-active substance, the rate of rise of the air bubbles is not affected. Stuke (9) ran experiments with oxygen bubbles rising in water containing small concentrations of caproic acid. The concentrations of the alcohols and the caproic acid were relatively small, hence the decrease in the surface tension was only about 1 dyne/cm. The authors conducted tests in water containing Glim, a liquid detergent. In the present study the concentration of Glim (0.42% by volume) was high enough to decrease the surface tension by 40 dynes/cm. No measurable change in the viscosity and density of the test liquid due to the presence of Glim was noted (see Table 1). This was also true for the alcohol and caproic acid solutions. Results from these experiments are presented in terms of the drag coefficient and Reynolds number in Figure 11.

The drag curve for bubbles in the Glim solution, as well as the experimental data from the other investigators in water containing at least the

10. It should be noted here that the physical properties of tap water did not differ from those of filtered water.

critical concentration of the surface-active substance, follows the drag curve of rigid spheres to a Reynolds number of about 200.[11] In the region of Reynolds numbers of 10 to 200, the drag curve for bubbles rising in a pure liquid having an M number very close to that of the Glim solution[12] follows the drag curve of fluid spheres. Thus, the motion of bubbles in water containing surface-active materials cannot be compared with that of bubbles in pure liquids on the basis of drag coefficient, Reynolds number, and M number, even in the region of ellipsoidal bubbles. Although the range of bubble sizes did not extend fully into the region of spherical cap bubbles, it is quite certain that the presence of surface-active substances will not alter their rate of rise, which was shown to be independent of all physical properties of the liquid.

The difference in behavior of the bubbles must be sought in the behavior at the surface. A high concentration of molecules of the surface-active substance will be found at the surface of the bubble. As for the case of the tap water, these molecules would travel with the bubble and impart, in effect, a 'rigid' surface to the bubble, that is to say impose the condition of zero velocity at the boundary.

Thus, the results of the tests given in Figure 11 show that these surface-active substances increase the drag of the bubbles (in the region of bubbles having equivalent radii of 0.03 to 0.30 cm); beyond the critical concentrations, any increase in concentration has relatively little influence on the drag of the bubbles.

Bubbles as Rigid Bodies. - In previous sections it was shown that bubbles rising in pure liquids [13] behave essentially like fluid bodies over a large range of bubble size, but that below a certain critical size (the size being different for various liquids) the bubbles behave like rigid bodies, that is to say the drag of the bubbles equals that of corresponding rigid bodies.

A possible explanation of the anomaly of behavior of the gas bubbles is given by Boussinesq's dynamic surface tension. He assumed that at interfaces in motion a dynamic surface tension exists. Its magnitude is given by the sum of the usual (static) surface tension and the dynamic increment. This increment includes a constant of proportionality (surface viscosity) which is a function of the two fluids composing the interface. As pointed out in the 'Introduction', for small bubbles the effect of the dynamic increment increases the drag to the value of corresponding rigid bodies. With increase in bubble size this effect becomes negligible and the drag of the bubble equals that of a fluid body. Therefore, for bubbles the transition region from 'rigid' to fluid bodies would be different for the various liquids. There is, however, no experimental evidence that dynamic surface tension, as postulated by Boussinesq, exists.

From a 'hydrodynamic' point of view the reason for the transition of the bubbles to 'rigid' bodies is not clear. The mere inclusion of surface tension as a pressure drop in the boundary conditions ($= 2\sigma/r$) does not alter the analytical solution for fluid spheres. Hence, it appears that the presence of surface tension should have no effect on the motion of the bubble, except in

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11. For concentrations below the critical, the drag curve lies between that of pure water and the curve shown in Figure 11 (9).
12. The 'M' number of Glim was 2.78×10^{-10}, that of Varsol, for example, was 4.3×10^{-10}.
13. Mixtures, such as the 13% ethyl alcohol-water mixture are included in this category.

maintaining the spherical shape. Thus the anomalous behavior of the bubbles cannot be explained in terms of 'hydrodynamics', but must be sought in terms of a surface phenomenon. If it could be shown analytically (if only in the region of slow flow) that equality of drag of corresponding rigid bodies and bubbles also implies equality of boundary conditions at the surface, then, as in the case of rigid bodies, the velocity on the entire surface of the bubble must vanish. The surface must then be able to hold molecules of the pure liquid, just as in the case of tap water and surface-active substances the surface attracts and holds a high concentration of particles or molecules of the surface-active substance. The molecules at the surface would travel with the bubble and hence would, in effect, give the same boundary conditions as a rigid surface. However, if the shear forces become larger in comparison to the forces holding the molecules at the surface, 'rigidity' at the surface cannot be maintained; circulation inside the bubble ensues and the drag of the bubble becomes smaller as compared to that of a rigid body.

Since it was not possible to correlate the results of the experiments on the motion of bubbles in the gravity pressure field in terms of non-dimensional parameters formed from the usual liquid properties (viscosity, surface tension, density), further work on freely rising bubbles is necessary before the results obtained from such tests can be utilized and before the more complicated behavior of bubbles in variable pressure gradients can be understood. Particularly, an understanding of the reason for the transition of bubbles from fluid to 'rigid' bodies as well as a criterion for this transition is most desirable, since such transition might be influenced by the magnitude of the pressure gradient. In the region of Reynolds numbers where the bubbles behave like rigid spheres, the pressure gradient probably has no effect on the drag coefficient.

SUMMARY

As the size of the bubbles was increased in the tests, a change in bubble shape from spherical to ellipsoidal to spherical cap shape was observed in all liquids. The volumes at which these transitions occur, however, varied with the properties of the liquid. For spherical bubbles of given volume the results show that the viscosity of the liquid is the most important property determining the rate of rise. For ellipsoidal bubbles the surface tension assumes greater importance. Spherical cap bubbles rise independently of the properties of the liquid.

The results show that the motion of air bubbles rising at their terminal velocity in a gravity field cannot be described completely by use of dimensionless parameters formed from the usual liquid properties (viscosity, surface tension, density), the equivalent radius of the bubble, the acceleration of gravity, and the terminal velocity.

The drag coefficients of tiny spherical bubbles coincide with those of corresponding rigid spheres. With increase in bubble size, a decrease in the drag as compared to that of rigid spheres occurs in some liquids. This change in the drag is due to the development of circulation inside the bubble. The drag curves of the spherical bubbles rising in various liquids fall between two limiting curves, namely the drag curves of rigid and fluid spheres, respectively. It was not possible to determine a criterion for the transition region of the bubbles from 'rigid' to fluid spheres.

Ellipsoidal bubbles occur for different ranges of Reynolds numbers for the various liquids. For liquids of low M number (say less than 10^{-3}) a

a minimum in the drag curve is reached at Reynolds numbers of the order of 250. These minima occur near the transition from spherical to ellipsoidal shape. Such minima in the drag curve are not obtained for liquids of high M number. For the liquids used, transition to spherical caps is completed at a Weber number of about twenty.

The drag coefficients of spherical cap bubbles are independent of bubble size and have a constant value of 2.6. The rate of rise of these bubbles as a function of the equivalent radius is given by the experimentally determined relation:

$$U = 1.02 \sqrt{g r_e}$$

For bubbles (ranging in equivalent radius from 0.03 to 0.25 cm) rising in tap water, an increased drag as compared to bubbles in clean (filtered or distilled) water was observed. The presence of certain surface-active substances in the water similarly increases the drag of bubbles (ranging in equivalent radius from 0.03 to 0.30 cm) as compared to bubbles in pure water. Beyond a certain critical concentration of these surface-active substances, an increase in concentration has relatively little influence on the drag of the bubbles.

Tests to determine the effect of the container walls on the velocity of rise indicate the absence of such effect for the range of bubble volumes and container sizes tested.

ACKNOWLEDGMENTS

The tests described in this paper were carried out at the David Taylor Model Basin, U. S. Navy.

The authors wish to acknowledge the suggestions of Dr. Lawrence M. Kushner of the National Bureau of Standards regarding the explanation of the behavior of bubbles in tap water and in water containing surface-active materials.

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APPENDIX II. LIST OF SYMBOLS

C_d	Drag coefficient $\left[= (8/3) g\, r_e / U^2\right]$
D	Drag of bubble
e	Coefficient of surface viscosity
g	Acceleration due to gravity
ℓ	Length parameter of bubble
M	Dimensionless parameter $\left[= \frac{g\mu^4}{\rho\sigma^3}\right]$
r_e	Equivalent radius of bubble, i.e. radius of a sphere of equal volume
R	Reynolds number $\left[= \frac{2 r_e U \rho}{\mu}\right]$
R'	Reynolds number inside bubble $\left[= \frac{2 r_e U \rho'}{\mu}\right]$
U	Terminal velocity
W	Weber number $\left[= \frac{2 r_e U^2 \rho}{\sigma}\right]$
μ	Coefficient of dynamic viscosity of liquid medium
μ'	Coefficient of dynamic viscosity of gas
ρ	Density of liquid medium
ρ'	Density of gas
σ	Surface tension

TABLE 1

Summary of Liquid Properties

Liquid	Temperature deg C	Viscosity μ poises	Density ρ gm/cc	Surface Tension σ dynes/cm	"M" $\frac{g\mu^4}{\rho\sigma^3}$
Water	19	0.0102	0.998	72.9	0.26×10^{-10}
Water	21	0.0098	0.998	72.6	0.24×10^{-10}
Cold Water	6	0.0147	0.999	74.8	1.08×10^{-10}
Hot Water	49	0.0056	0.989	68.1	0.307×10^{-11}
Glim Solution	19	0.0103	1.000	32.8	2.78×10^{-10}
Mineral Oil	27.5	0.580	0.866	20.7	1.45×10^{-2}
Varsol	28	0.0085	0.782	24.5	4.3×10^{-10}
Turpentine	23	0.0146	0.864	27.8	24.1×10^{-10}
Methyl Alcohol	30	0.0052	0.782	21.8	0.89×10^{-10}
62 percent Corn Syrup and Water	22	0.550	1.262	79.2	0.155×10^{-3}
68 percent Corn Syrup and Water	21	1.090	1.288	79.9	0.212×10^{-2}
56 percent Glycerine and Water (Bryn)	18	0.0915	1.143	69.9	1.75×10^{-7}
42 percent Glycerine and Water (Bryn)	18	0.043	1.105	71.1	4.18×10^{-8}
13 percent Ethyl Alcohol and Water (Bryn)	22	0.0176	0.977	43.5	1.17×10^{-8}
Olive Oil (Arnold)	22	0.73	0.925	34.7	0.716×10^{-2}
Syrup (Bond)	17	180	1.48	91	0.92×10^{6}

TABLE 2

Bubble Shapes as Function of Reynolds Number

Liquid	Reynolds Number		
	Spherical	Ellipsoidal	Spherical Cap
Water	less than 400	400 - 5000	larger than 5000
Cold water	275	275 - 3000	3000
Mineral Oil	0.45	0.45 - 80	80
Varsol	80	80 - 2000	2000
Turpentine	85	85 - 1500	1500
Methyl Alcohol	80	80 - 4000	4000
62% Corn Syrup and Water	0.28	0.28 - 60	60
68% Corn Syrup and Water	2.5	2.5 - 110	110

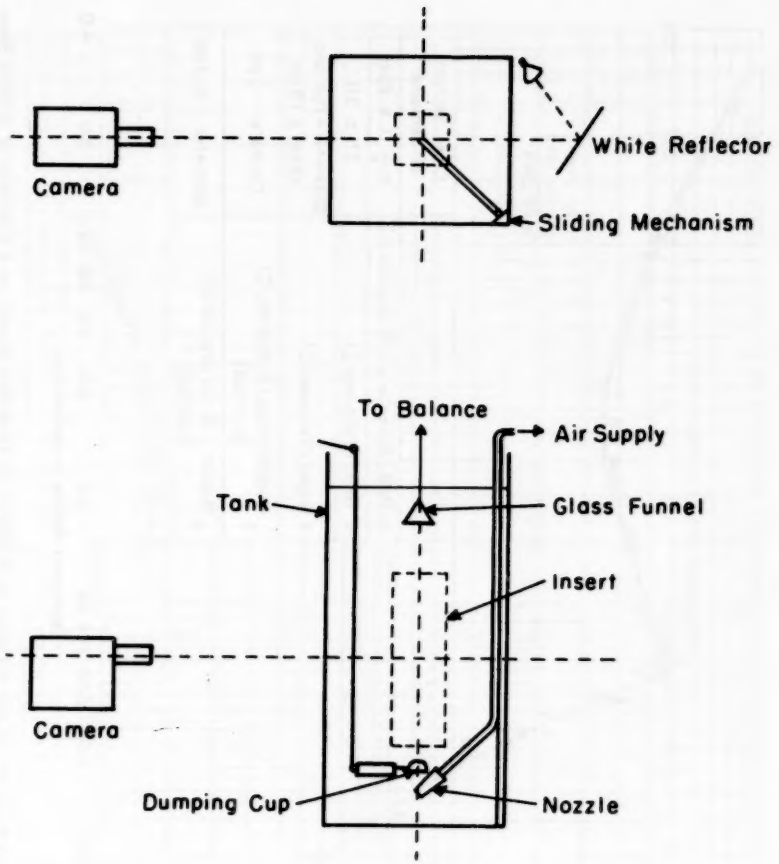


Fig. 1. Experimental Setup

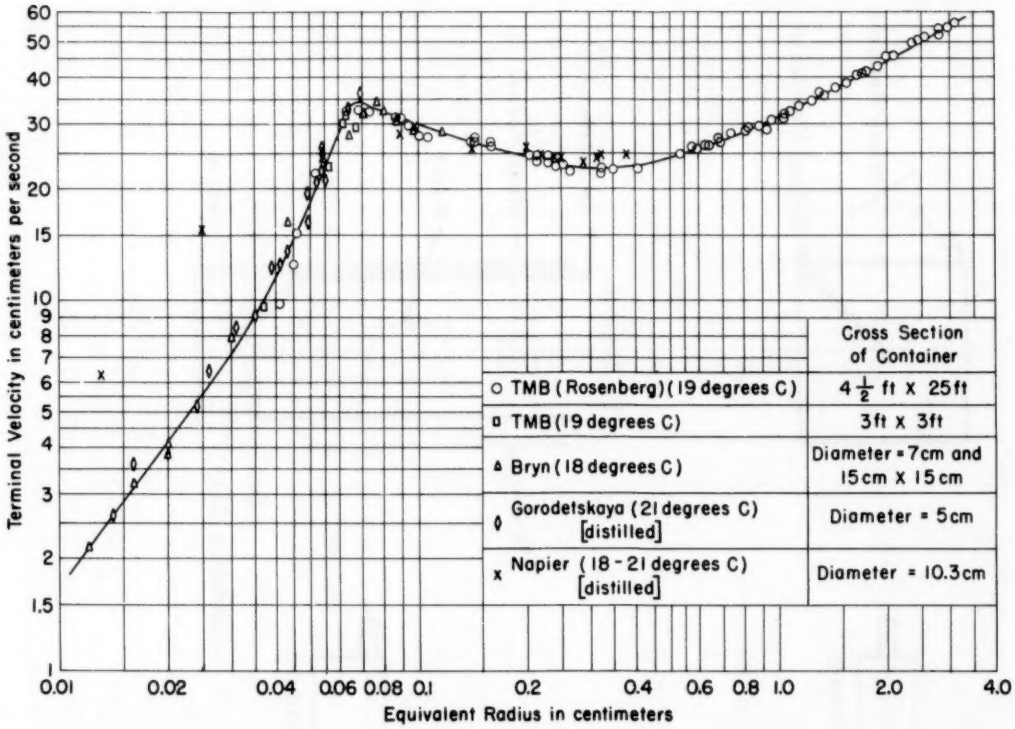


Fig. 2. Terminal Velocity of Air Bubbles in Filtered or Distilled Water as a Function of Bubble Size

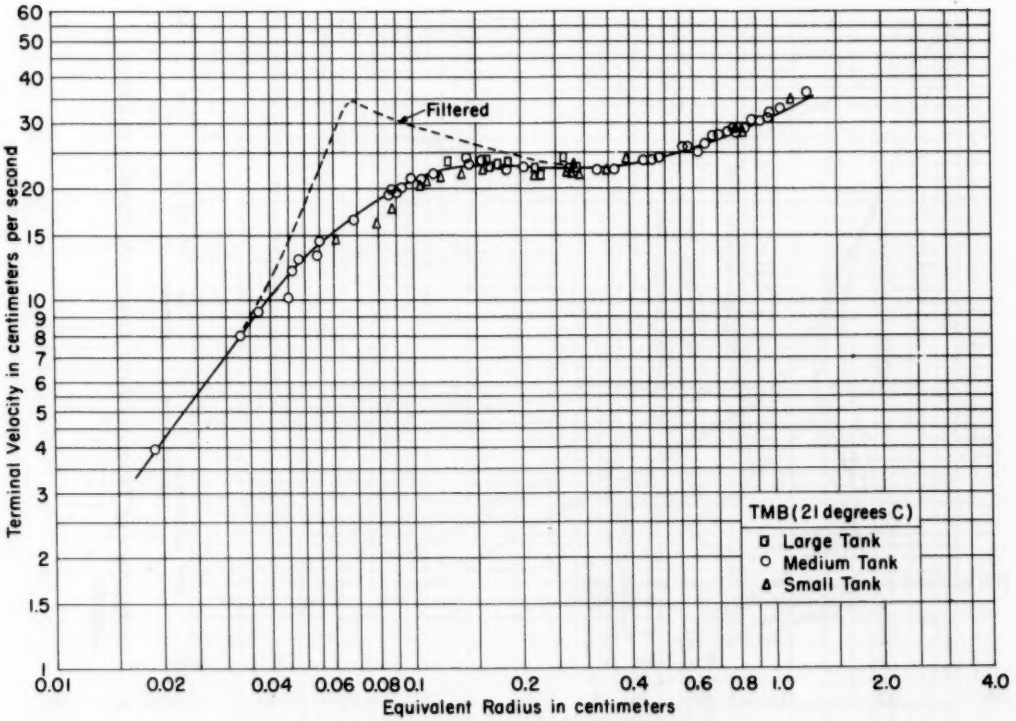


Fig. 3. Terminal Velocity of Air Bubbles in Tap Water as a Function of Bubble Size

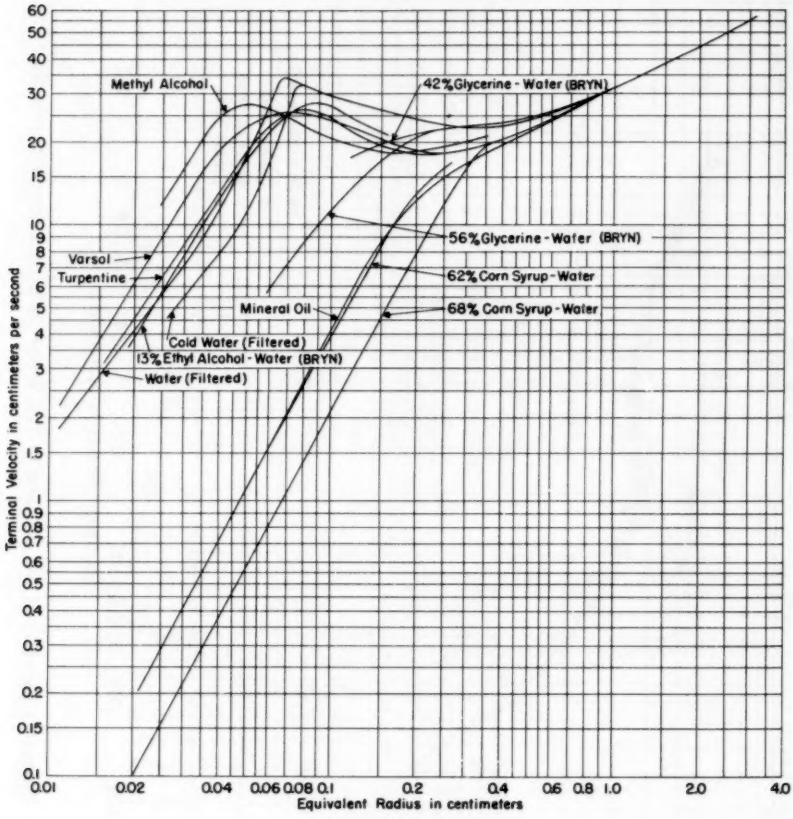


Fig. 4 Comparison of Terminal Velocities of Air Bubbles in Various Liquids

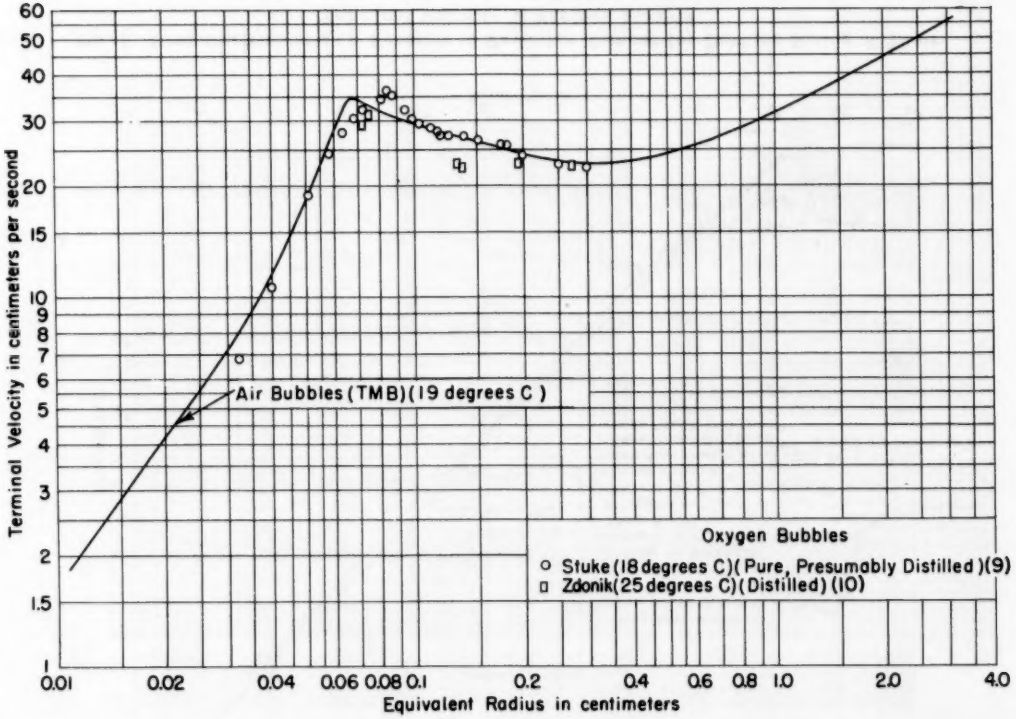


Fig. 5. Terminal Velocity of Gas Bubbles in Distilled Water as a Function of Bubble Size

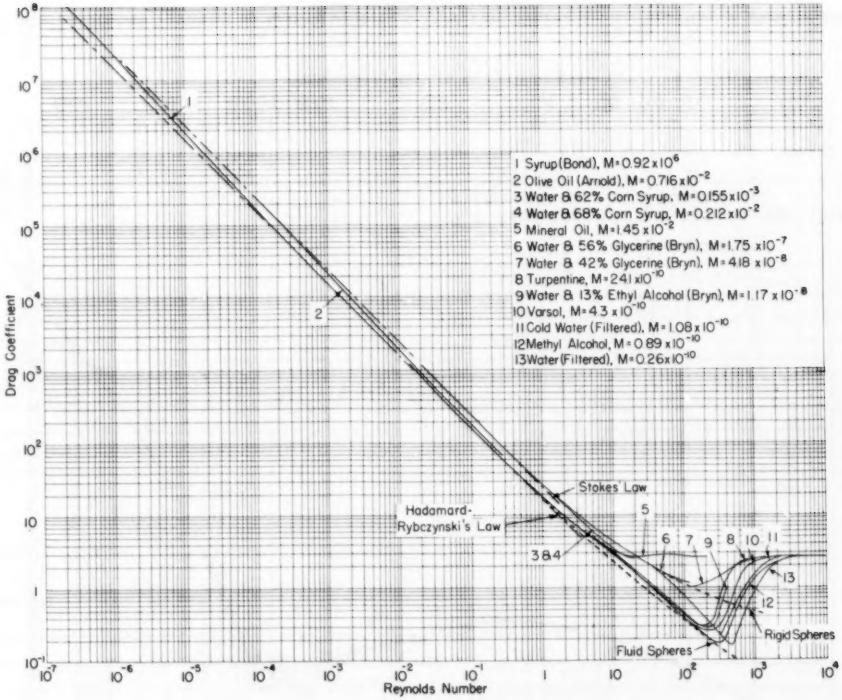


Fig. 6. Drag Coefficient as a Function of Reynolds Number for Bubbles Rising at Their Terminal Velocity in Various Liquids

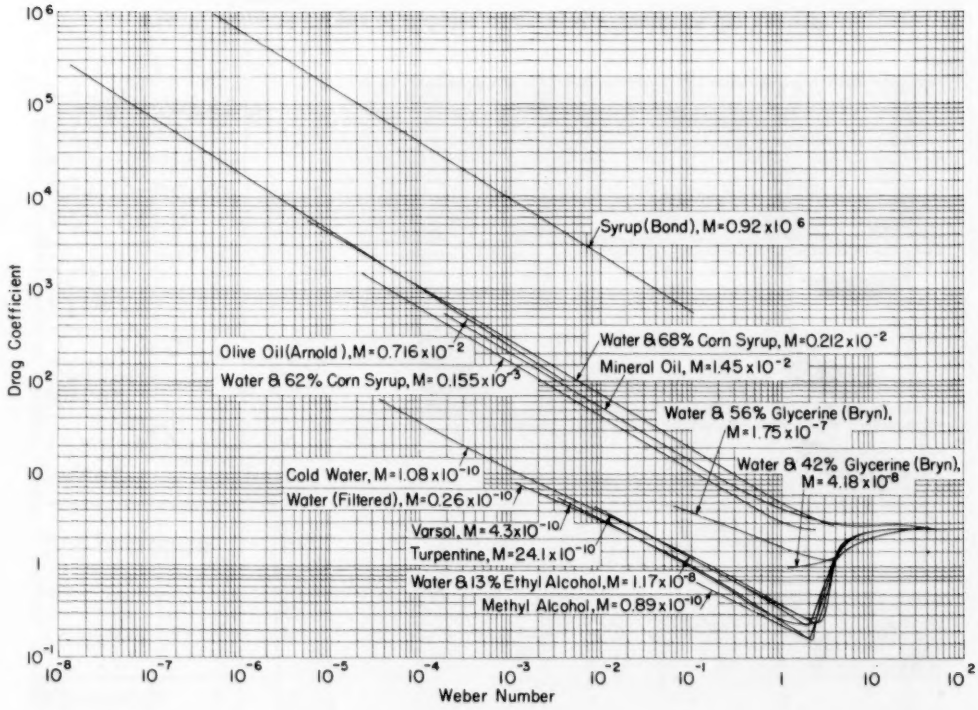


Fig. 7. Drag Coefficient as a Function of Weber Number for Bubbles Rising at Their Terminal Velocity in Various Liquids

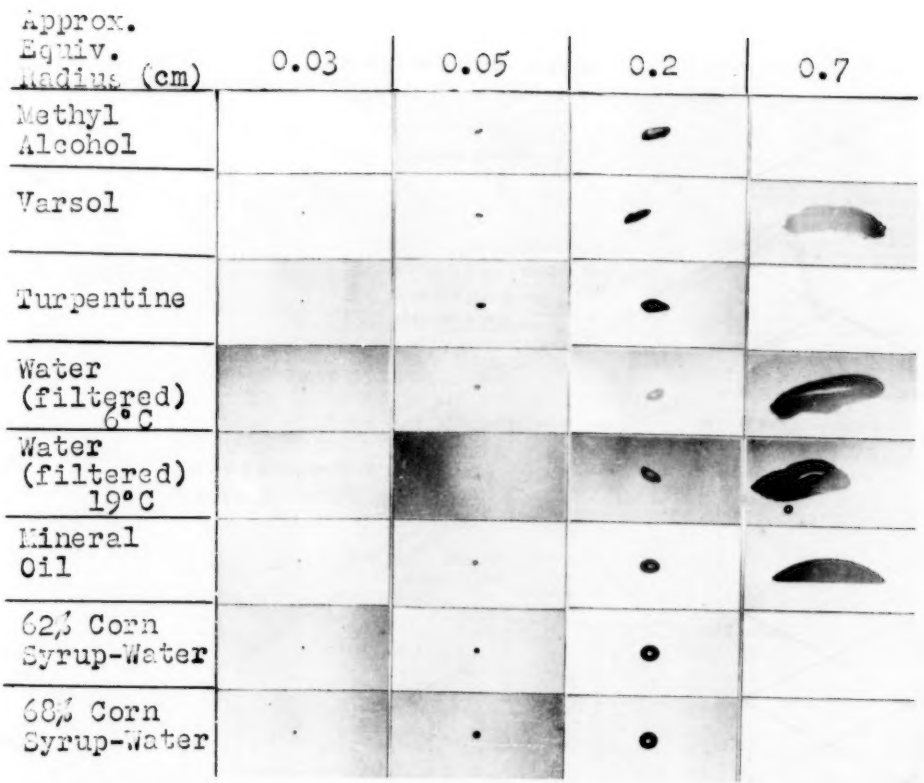
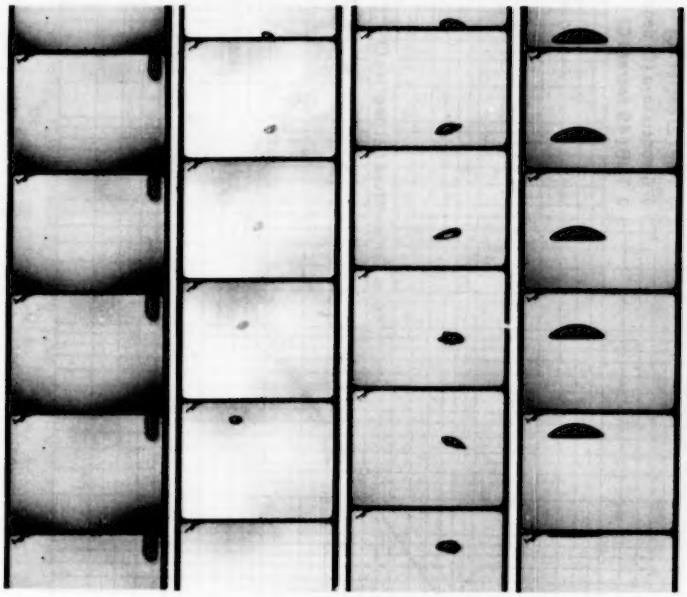


Fig. 8. Typical Shapes of Bubbles of Several Volumes in the Various Liquids



Liquid	VARSOL	COLD WATER (filtered)	METHYL ALCOHOL	MINERAL OIL
Equiv. Radius (cm)	0.033	0.190	0.317	0.570
Velocity (cm/sec)	13.3	23.5	20.4	22.7
Reynolds Number	81	628	1960	42

Fig. 9. Typical Paths of Bubbles Rising in Various Liquids

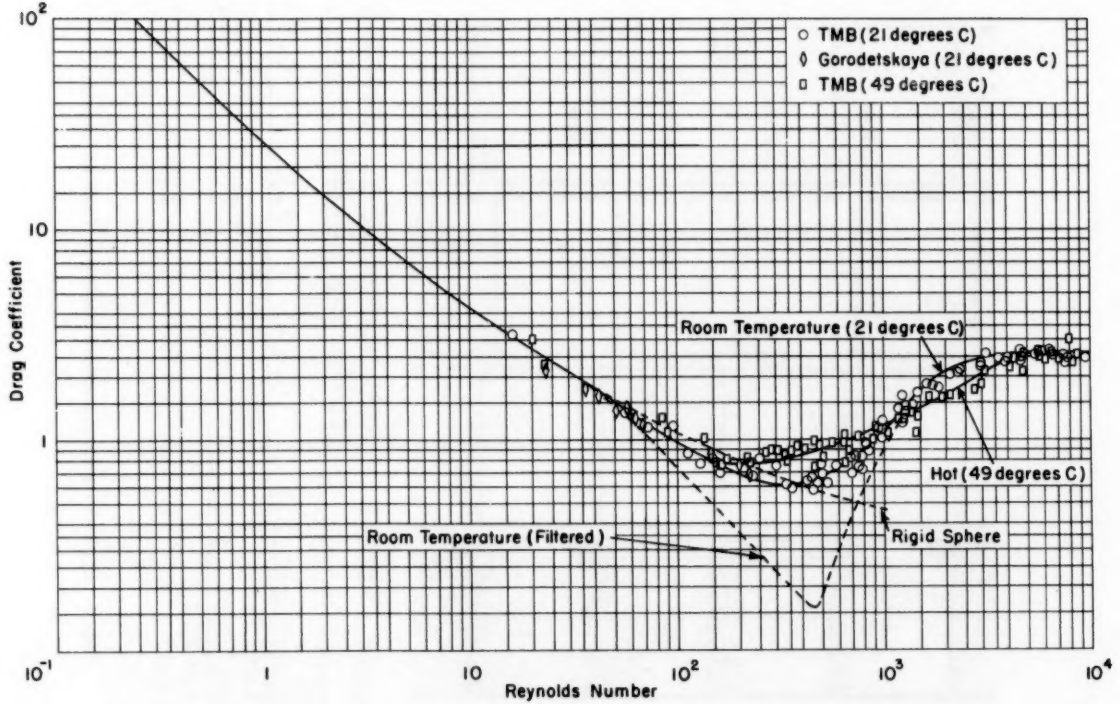


Fig. 10. Drag Coefficient as a Function of Reynolds Number for Bubbles Rising at Their Terminal Velocity in Tap Water

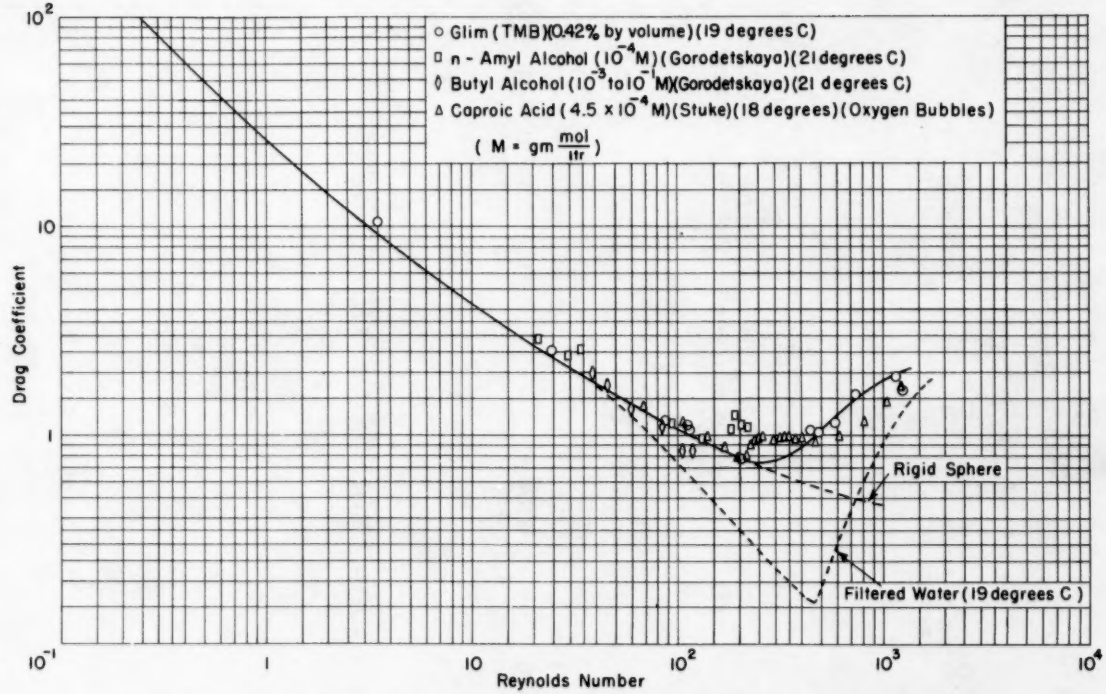


Fig. 11. Drag Coefficient as a Function of Reynolds Number for Bubbles Rising at Their Terminal Velocity in Water Containing Various Surface-Active Materials

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